Function using FOPDT, SOPDT and SKOGESTAD in Control System

Raghdah Bakery Khader Elssadig¹ and Dr. Eltahir Mohammed Hussien²

¹Faculty of Engineering, Al Neelain University, Khartoum, Sudan
²Collage of Engineering, Sudan University of Science and Technology (SUST), Khartoum, Sudan

Abstract
Working with high order transfer function needs a lot of work. Transfer functions of high order may be approximate as low order transfer function with time delay. This done by First Order Plus Dead Time (FOPDT), Second Order Plus Dead Time (SOPDT) and Skogestad. The main objective of this research is to approximate the higher order transfer function to first and second order transfer function using the three methods. Then compare the result with the original plant transfer function so as to test the dependency of these methods. DC motor speed transfer function was taken as model to verify the result. The main objectives of this paper are Study and investigate the response of higher order transfer function, approximate the higher order transfer function to lower order using FOPDT, SOPDT and Skogestad, show that the approximated lower transfer function gives the same response of original higher order transfer functions, test the degree of dependency of these methods in control system, implement FOPDT, SOPDT and Skogestad on DC motor position transfer function.

Keywords: FOPDT, SOPDT, SKOGESTAD.

1. Introduction

Approximation theory is a branch of mathematics, a quantitative part of functional analysis. Approximation usually occurs when an exact form or an exact numerical number is unknown or difficult to obtain. To develop a mathematical model for a process is often the first step undertaken in design of a controller. It has been recognized that a first or second order plus dead time. Identification methods have been reported and they are generally classified into parametric and non-parametric approaches. Transfer functions might be the most the welcome parametric model. Process models described by transfer functions play a vital role in process analysis, control and optimization. To obtain a transfer function description of a process, identification methods may be sorted into two categories, open loop types and closed-loop types. In earlier years, the first and second order plus dead-time model of the process was estimated from the process reaction curve obtained from an open-loop step response of the process, with the risk of process runaway. FOPDT or SOPDT model approximation to a complicated process system can be carried out system through either a kind of model reduction approach or a kind of system identification approach.

2. Methodology

Working with high order TF needs a lot of work. Transfer functions of high order, may be approximate as low order transfer function. The most commonly used model to describe the dynamics process is the First Order Plus Time Delay model, Second Order Plus Time Delay and Skogestad methods. Time delay or Dead Times (DT) between input and output are very common in industrial processes and engineering system. Transportation and measurements lag, analysis time, computation and communication lags all introduce DT into control loop. Time Delay are also used to compensate for model reduction where high order system are represented by low order model with delay.

2.1 First Order Plus Dead Time (FOPDT)

First Order Plus Dead Time
\[ W(s) = \frac{K}{1 + T \cdot s} e^{-\tau \cdot s} \]  

.................................................. (1)

Where:

K - Process gain: the ultimate value of the response (new steady state) for a unit step change in the input.

T - Time constant: measure of time needed for the process to adjust to a change in the input.

\( \tau \) - Delay: the time at which output of the system begins to change minus the time at which the input step change was made [3].

- Three parameters: K; T; \( \tau \) simple but moderately complex
- Describes the dynamics of the system with sufficient accuracy controller are working on their basis
- Easily obtained with simplification of the complex models
- Easy to identify

The problem: if the highest insignificant time constant (T2) is close to the important time constant (T1) [3,6]. The steps to approximate the transfer function to FOPDT as follow:

1. Arrange the high order system in factorial

\[ \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)(\tau_4 s + 1)} \]  

..................................................(2)

2. Assuming \( \tau_1 \) is the biggest

\[ = \frac{K_p}{\tau_1 s + 1} \cdot e^{-\tau_2 s} \cdot e^{-\tau_3 s} \cdot e^{-\tau_4 s} \]  

\[ = \frac{K_p e^{-(\tau_2 + \tau_3 + \tau_4)s}}{\tau_1 s + 1} \]  

..................................................(3)

Figure 1: First orders plus time delay response to step change
2.2 Second Order PLUS DEAD TIME (SOPDT)

The second order plus dead time may be express as:

\[ W(s) = \frac{K e^{-\tau s}}{(1 + T_1 \cdot s)(1 + T_2 \cdot s)} \]  

The steps to approximate the transfer function to SOPDT as follow:

1. Arrange the high order system in factorial as

\[ \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)(\tau_4 s + 1)} \]  

2. \( \tau_1, \tau_2 \) are the largest

3. An approximate SOPDT model would be

\[ \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)(\tau_4 s + 1)} z^{-|z-1|} \]  

Figure 2: Step response for SOPDT
2.3 Skogestad method

Skogestad has proposed a related method for FOPDT and the transfer function will be:

\[ \frac{K_p}{(\tau_i s + 1)(\tau_d s + 1)(\tau_s s + 1)} \approx K_p e^{\frac{\tau_d}{2} s + \frac{\tau_d}{4}} \]

……………………………..……..(7)

Skogestad has proposed a related method for SOPDT and the transfer function will be:

\[ \frac{K_p}{(\tau_s s + 1)(\tau_i s + 1)(\tau_d s + 1)} \approx K_p e^{\frac{\tau_d}{2} s + \frac{\tau_d}{4}} \]

……………………………....(8)

Figure 3: Step response of Skogestad with FOPDT of DC motor speed

3. Result and Discussion

For higher order and complicated systems the designer will be concerned with the analysis and design of the control system. For example when choosing the parameter of the PID controller and for testing the system performance, will no longer being using the actual system but its corresponding or equivalent FOPDT or SOPDT instead. Modern control such as fuzzy control and neuron network doesn’t take consideration for the model itself and its transfer function. This higher level of control system approach, now using FOPDT and SOPDT instead of transfer function, Could be considered as another high level or (midpoint) of control system because It doesn’t deal with complicated transfer function, it deals with very simple equivalent transfer function, the analysis and the design of controller will be easier, the system performance measurement is simple and finally, it is decrease the cost. FOPDT transfer function is first order transfer function so it is very easy to analysis and design. Because of that using FOPDT of DC motor speed to choose the parameter of PID controller that represent the same parameter of the original transfer function dc motor speed. The step response of FOPDT with PID controller is similar to step response of DC motor speed with the same controller.
4. Conclusion

As higher order systems are so complicated in analysis and control, the thesis proposed a simple approach to replace the higher order complicate system with a simple equivalent system. The simple equivalent transfer function was approximated using FOPDT and SOPDT methods. The main benefits of using such way that when choosing the controller parameters, it was much easier to determine their response with FOPDT and SOPDT. The result as show that the controller deducted by FOPDT or SOPDT was the controller for the original system.

References


