Combined Sliding Mode Control with a Feedback Linearization for Speed Control of Induction Motor

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Abstract—Induction Motor (IM) speed control is an area of research that has been in prominence for some time now. In this paper, a nonlinear controller is presented for IM drives. The nonlinear controller is designed based on input-output feedback linearization control technique, combined with sliding mode control (SMC) to obtain a robust, fast and precise control of IM speed. The input-output feedback linearization control decouples the flux control from the speed control and makes the synthesis of linear controllers possible. To validate the performances of the proposed control scheme, we provided a series of simulation results and a comparative study between the performances of the proposed control strategy and those of the feedback linearization control (FLC) schemes. Simulation results show that the proposed control strategy scheme shows better performance than the FLC strategy in the face of system parameters variation.

I. INTRODUCTION

DC motors have been used extensively in the industry mainly because of the simple control techniques required to achieve good performance in speed or position control applications. However, in comparison with their counterparts, the IM drives, DC motor drives result more expensive and less robust devices, not to mention the periodic maintenance they require due to the commutator. In spite of the superiority of IM drives over DC ones, they were rarely used in control applications in the past since they are described by nonlinear models from which poor performance control schemes resulted. In the last few decades, abundant research and development efforts for IM control technology have been made. The most popular high performance IM control technique is known as vector control (VC), proposed by Hasse and Blaschke. VC of induction motor achieves decoupled torque and flux dynamics leading to independent control of the torque and flux as for a separately excited DC motor [1]. However, the performance is degraded due to motor parameter variations and unknown external disturbances [2]. FLC design is one of the most widely used nonlinear approaches to the control problem, which has attracted a great deal of research interest in recent years. The central idea of the approach is to algebraically transform a nonlinear system dynamics into a (fully or partly) linear one, so that linear control techniques can be applied. FLC has been used successfully to address some practical control problems. These include the control of high performance aircraft, helicopters, industrial robots, and IM. More applications of the methodology are being developed in industry. However, there are also a number of important shortcomings and limitations associated with the FLC approach. Such as sensitive to the parameter variations and external disturbances [3]. In the recent past years, the SMC strategies have received worldwide interest, and many theoretical studies and application researches are reported [4]. It is known that the SMC can offer such properties as insensitivity to parameters variations, external disturbance rejection, fast dynamic response, and simplicity of design and implementation. To overcome the above problems and achieve accurate control performance of speed control of IM, a robust controller which is design by employing SMC and FLC.

In this paper, we will discuss the basic concepts of FLC and apply this technique to speed control of the IM. Then, the combined between SMC with FLC is presented to rotor speed control of IM. To validate the performances of the proposed control law, we provided a series of simulation results and a comparative study between the performances of the proposed control strategy, and FLC under two different test conditions to show the control properties.

II. BASIC CONCEPT OF FLC

The theory of feedback linearization control has been represented in many papers [6-8]. The main mathematics method is based on the differential geometry or the Lie derivation. If we consider the following multi-input multi-output (MIMO) system presented in (1-2).

\[ x' = f(x) + \sum_{j=1}^{m} g_j u_j \]

Where \( x \) is the \( n \times 1 \) state vector, \( u \) is the \( m \times 1 \) control input vector, and \( f(x) \) and \( g_j \) are smooth vector fields. Then, \( m \) outputs have to be chosen in order to get a square system.

\[ y_i = h_i(x) \quad 1 \leq i \leq m \]

Where \( y_i \) is the \( m \times 1 \) vector of system outputs, and \( h_i(x) \) is smooth vector fields. Now we will introduce some concepts that would help understand the development of the feedback linearization control law.

A. Relative Degree

The partial relative degrees \( r_i \) is equal to the number of times the output \( y_i \) has to be differentiation until at least one input appears in the derivative, and the total relative degree of the system defined by \( r = \sum_{j=1}^{m} r_j \).
B. Lie Derivatives

The Input-output linearization of the above MIMO systems is obtained by differentiation of each output r times until at least one input appears in the derivative. If we take the first derivative of the first output $y_1$, we define the following notation.

$$ \dot{y}_1 = L_f h_1 (x) + \sum_{j=1}^{m} L_{gj} h_1 (x) u_j $$

(3)

Where $\sum_{i=1}^{n} \frac{\partial h_i}{\partial x_i} f_j (x) = L_f h_i (x)$ is called a Lie derivative. If the relative degree $r_1$ is larger than 1 we have that $L_{g1} h_i (x) = \ldots = L_{gm} h_i (x) = 0$. We have to repeat this process until we find that $L_f L_f h_i (x) \neq 0$ for at least one $j$. For convenience the following notation is mostly used.

$$ L_g L_f h_i (x) = \frac{\partial (L_f h_i (x))}{\partial x} g_i (x) $$

(4)

$$ L_f h_i (x) = L_f L_f h_i (x) = \frac{\partial (L_f h_i (x))}{\partial x} f_i (x) $$

(5)

$$ L_f h_i (x) = h_i (x) $$

(6)

After performing the procedure for each output, we are left with the m equations corresponding to the m outputs.

$$ \begin{bmatrix} y_1^r \\ \vdots \\ y_m^r \end{bmatrix} = L_f h_i (x) + D(x) u_i $$

(7)

Where the $m \times m$ matrix $D(x)$ is called the decoupling matrix for MIMO system and defined as:

$$ D(x) = \begin{bmatrix} L_g L_f h_1 (x) & \ldots & L_g L_g h_1 (x) \\ \vdots & \ddots & \vdots \\ L_g L_g h_m (x) & \ldots & L_g L_g h_m (x) \end{bmatrix} $$

(8)

As long as $D(x)$ is non-singular, then the linearizing control law $u$ can be obtained by:

$$ \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = -D(x)^{-1} \begin{bmatrix} \dot{y}_1 \\ \vdots \\ \dot{y}_m \end{bmatrix} + D(x)^{-1} \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} $$

Where $[v_1 \ldots v_m]^T$ is the new set of inputs defined by the designer. By substitution of (9) into (7), we can cancel the nonlinearities and obtain the simple input-output relation is given by:

$$ \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} $$

(10)

It is readily noticed that the input-output relation in equation (10) is decoupled and linear since the input $v$ only affects the output $y$. To ensure perfect tracking, the new control signal $[v_1 \ldots v_m]^T$ is defined as:

$$ v_i = y_i^r - h_i (x) - k_i e_i - k_{i-1} e_{i-1} - \ldots - k_0 e_0 $$

(11)

Where $e = y - y_{ref}$ is the tracking error. By choosing the coefficients $k_i$ so that the polynomial $p_i (\lambda) + k_i \lambda^{r_i - 1} + \ldots + k_0 \lambda^0$ has all its roots strictly in the left-half complex plane, both the convergence to zero of error function and the overall stability of the system are guaranteed.

III. INDUCTION MOTOR MODEL

Under the assumptions of linearity of the magnetic circuit, equal mutual inductances, and neglecting iron losses, a three-phase squirrel-cage induction machine model in the fixed stator d-q reference frame can be described as a fifth order nonlinear differential equations with four electrical variables (stator currents ($i_{ds}$, $i_{qs}$) and rotor fluxes ($\psi_{ds}$, $\psi_{qs}$)), and one mechanical variable (rotor speed $\omega_R$) [9-10].

$$ \dot{x} = f (x) + g_1 i_{ds} + g_2 i_{qs} $$

(12)

Where

$$ g_1 = \frac{1}{L_s} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad g_2 = \frac{1}{L_o} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T $$

(9)

$$ x = [x_1 x_2 x_3 x_4 x_5]^T = [i_{ds} i_{qs} \psi_{ds} \psi_{qs} \omega_R]^T $$

$$ \alpha = \frac{R_s}{L_s}, \quad \beta = \frac{R_s R_m}{L_s L_r}, \quad \gamma = \frac{L_r}{L_s L_o} $$

Where $f(x)$ is the nonlinear function:

$$ f(x) = \begin{bmatrix} -\alpha i_{ds} + \beta \psi_{dr} + \omega_R \gamma \psi_{qr} \\ -\alpha i_{qs} + \beta \psi_{qr} - \omega_R \gamma \psi_{dr} \\ \delta i_{ds} - T_m \psi_{dr} - \omega_R \psi_{qr} \\ \delta i_{qs} - T_m \psi_{qr} + \omega_R \psi_{dr} \\ P \frac{3P}{2} \left( \psi_{dr} i_{qs} - \psi_{qr} i_{ds} - \frac{T_L}{4L_J} \right) \end{bmatrix} $$

$$ L_g = \frac{1}{L_s} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad g_2 = \frac{1}{L_o} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T $$

(9)

$$ x = [x_1 x_2 x_3 x_4 x_5]^T = [i_{ds} i_{qs} \psi_{dr} \psi_{qr} \omega_R]^T $$

$$ \alpha = \frac{R_s}{L_s}, \quad \beta = \frac{R_s R_m}{L_s L_r}, \quad \gamma = \frac{L_r}{L_s L_o} $$

Where $f(x)$ is the nonlinear function:
\[
L_a = L_s - \frac{L_m}{T_f}, \quad \delta = \frac{L_m}{T_i}, \quad T_i = \frac{1}{T_r}, \quad d = \frac{3\pi^2 L_m}{8L_J}
\]

Where \(L_c\) is the stator inductance, \(L_s\) is the rotor inductance, \(L_m\) is the mutual inductance, \(L_a\) is the redefined leakage inductance. \(R_s\) and \(R_r\) are stator and rotor inductance resistances, respectively. \(J\) is the moment of inertia of the motor, \(T_e\) is the torque of external load disturbance, \(P\) is the number of pole, and \(T_i\) is the time constant of the rotor dynamics. From (12) the rotor speed is a nonlinear output with respect to the state variables of the dynamical model. Therefore, it is difficult to control the rotor speed directly from control inputs \(v_{ds}\) and \(v_{qs}\)

IV. COORDINATE TRANSFORMATIONS

In this section the objective of the proposed control scheme is to control independently the rotor speed and the square of the rotor flux to follow the desired control signals. To fulfill the control objectives, the differential geometry and nonlinear feedback linearization techniques are used to transform the original motor model in (12) to a coordinate transformed model which decouples the control of rotor speed and the square of the rotor flux. In order to control rotor speed and the square of the rotor flux, the rotor speed and the square of the rotor flux are chosen as the outputs of the coordinate model. The output equation is:

\[
y = \begin{bmatrix}
y_1(x) \\
y_2(x) \\
y_3(x) \\
y_4(x)
\end{bmatrix} = \begin{bmatrix}
y_{1}(x) \\
y_{2}(x) \\
y_{3}(x) \\
y_{4}(x)
\end{bmatrix} = \begin{bmatrix}
y_{1}(x) \\
y_{2}(x) \\
y_{3}(x) \\
y_{4}(x)
\end{bmatrix}
\]

Define the change of coordinates as:

\[
z_1 = \frac{1}{T_f}
\]

\[
z_2 = \frac{1}{2}
\]

\[
z_3 = \frac{1}{2}
\]

\[
z_4 = \frac{1}{2}
\]

Then, the dynamic model of the induction machine is given in new coordinate by:

\[
\dot{z}_1 = \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
\dot{z}_2 = \begin{bmatrix}
0 \\
L_g L_f h_2(x) \\
L_g L_f h_2(x) \\
\end{bmatrix}
\]

\[
\dot{z}_3 = \begin{bmatrix}
0 \\
L_g L_f h_2(x) \\
L_g L_f h_2(x) \\
\end{bmatrix}
\]

\[
\dot{z}_4 = \begin{bmatrix}
0 \\
L_g L_f h_2(x) \\
L_g L_f h_2(x) \\
\end{bmatrix}
\]

This system model is written in terms of higher derivatives of output \(y_1\) and \(y_2\) as follows:

\[
\begin{bmatrix}
\dot{y}_1(x) \\
\dot{y}_2(x)
\end{bmatrix} = \begin{bmatrix}
L_f^2 h_1(x) + D(x)[v_{ds}] \\
L_f^2 h_2(x) + D(x)[v_{qs}]
\end{bmatrix}
\]

The decoupling matrix \(D(x)\) is defined as:

\[
D(x) = \begin{bmatrix}
L_g L_f h_1(x) \\
L_g L_f h_2(x)
\end{bmatrix}
\]

Where:

\[
L_f^2 h_1(x) = \left( \frac{ad + \frac{d}{T_f}}{b_1} \right) - \frac{d}{T_f} b_2 - \frac{d}{T_f} b_3
\]

\[
L_f^2 h_2(x) = -\frac{d}{T_f} b_4 - \frac{2a}{T_f} b_5 + \frac{2d}{T_f} b_6
\]

The decoupling matrix \(D(x)\) is singular if and only if the square of the rotor flux is zero which only occurs at the start up of the motor. That is, to fulfill this condition one can use in a practical setting, an open loop controller at the start up of the motor, and then switch to the nonlinear controller as soon as the flux goes up to zero. If the decoupling matrix is not singular, the nonlinear state feedback control is given by:

\[
\begin{bmatrix}
v_{ds} \\
v_{qs}
\end{bmatrix} = D(x)^{-1} \begin{bmatrix}
L_f^2 h_1(x) + v_1 \\
L_f^2 h_2(x) + v_2
\end{bmatrix}
\]

The controller linearizes and decouples the system resulting in:

\[
\begin{bmatrix}
\dot{y}_1(x) \\
\dot{y}_2(x)
\end{bmatrix} = \begin{bmatrix}
\frac{d}{T_f} b_1 \\
\frac{d}{T_f} b_2
\end{bmatrix}
\]

To ensure perfect tracking of rotor speed and the square of the rotor flux references, respectively. The variables \(v_1\) and \(v_2\) are chosen as follows:
\[ v_1 = -k_1 \left( \omega - \omega_{ref} \right) - k_2 \left( \psi - \psi_{ref} \right) \]
\[ v_2 = -k_3 \left( \psi^2 - \psi_{ref}^2 \right) - k_4 \left( \psi^2 - \psi_{ref}^2 \right) \]  
(20)

Where \( k_1, k_2, k_3, \) and \( k_4 \) are positive non-zero constants to be determined in order to make the closed loop system stable and to have fast response in variable tracking. Now, a linear reference model is used to set the desired output dynamic behaviours of the nonlinear system as follows:

\[
\begin{align*}
\dot{h}_{m1}(x) &= 0 \quad 1 \quad 0 \quad 0 \\
\dot{h}_{m2}(x) &= -a_1 \quad -a_2 \quad 0 \quad 0 \\
\dot{h}_{m3}(x) &= 0 \quad 0 \quad 1 \quad 0 \\
\dot{h}_{m4}(x) &= 0 \quad 0 \quad -a_3 \quad -a_4
\end{align*}
\]  
(21)

Where \( a_1, a_2, a_3, \) and \( a_4 \) are positive constants determining the rotor speed dynamic and rotor flux dynamic performances, respectively. The control algorithm consists of two steps:

(i) Calculation \( \left[ v_1 \ v_2 \right] \) according to (20).

(ii) Calculation \( \left[ v_{d1} \ v_{q1} \right] \) according to (18).

V. SIMULATION RESULTS

To evaluate the performance of the proposed control technique, we provided a series of simulations and a comparative study between the performances of the proposed control strategy and FLC under two different test conditions, nominal inertia, and rotor resistance mismatch. The two control method schemes are compared using the same rotor speed reference command. The rotor speed command is changed from 5rad/sec to 50rad/sec. The parameters of the linear reference model are selected such that rotor speed rise time is 0.2sec with no overshoot and the reference rotor flux command is set to 0.567wb. The specification of the induction motor system is given in Appendix. Simulation tests are carried out using Matlab/Simulink software package. The simulation is carried out based on the scheme shown in Figure 1.

A. Nominal Condition

In this section the tracking performances of the proposed control technique and FLC schemes are compared under nominal condition. Figures 2-4 show the rotor speed tracking, rotor speed tracking error, and stator current using the proposed control technique scheme, respectively. Figures 5-7 show the rotor speed tracking, rotor speed tracking error, and stator current results from the FLC scheme with the same rotor speed and rotor flux reference commands, respectively. The results show that similar control performance is obtained using the proposed control strategy and FLC schemes. However, it shows that the FLC scheme has a greater transient rotor speed error compared to the proposed control strategy scheme. This means that during the rotor speed transients the proposed control strategy scheme can track the rotor speed command more accurately than the FLC scheme.

Fig. 2 Rotor speed tracking performance using SMCFLC

Fig. 3 Rotor speed tracking error using SMCFLC

Fig. 4 Stator current \( i_{qs} \) using SMCFLC
B. Increase the Rotor Resistance

In order to test the robustness of the controller schemes with rotor resistance uncertainty, the rotor resistance is stepped to 1.75Ω during the simulation tests. Figures 8-10 are the rotor speed tracking, rotor speed tracking error, and stator current using the proposed control strategy scheme. The rotor speed dynamics does not alter when the rotor resistance is increased. The result shows that the proposed control strategy scheme is robust to rotor resistance uncertainty, with smaller peak rotor speed error. Figures 11-13 show the rotor speed tracking, rotor speed tracking error, and stator current using the FLC scheme. The rotor speed response is slower than the proposed control strategy scheme due to the rotor resistance uncertainty present in the system. This is because less torque is created with the increased rotor resistance than the case with accurate rotor resistance. It is noted that the value of stator current again increases during the rotor speed transient, which is much higher than the value of stator current under nominal condition. The reason for this is that a high stator current is needed to create more torque to make the rotor speed change faster.
VI. CONCLUSIONS

In this paper, the methodology combined sliding mode control and feedback linearization control is submitted to design the rotor speed control of induction motor. To evaluate the performance of the proposed control strategy scheme, we provided a series of simulation tests and a comparative study between the performances of the proposed control strategy and those of the feedback linearization control schemes. From the comparative simulation results, one can conclude that the two methods techniques demonstrate nearly the same dynamic behaviour under nominal condition. Robustness of the two method techniques against system parameters variation is also verified. Simulation results show that the proposed control strategy scheme shows better performance than the feedback linearization control strategy in the face of system parameters variation.

VII. APPENDIX

Table I Electrical and mechanical parameters of the IM

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<th>Parameters</th>
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<td>Stator phase</td>
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<td>Rated power</td>
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<td>Line voltage</td>
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<td>Line current</td>
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<td>Rated torque</td>
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<tr>
<td>Stator inductance, $L_s$</td>
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REFERENCES