OFDM Based on Complex wavelet Transform

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is normally implemented using a Fourier Transform (FT) to create and detect the different subcarriers. This can be efficiently implemented using an Inverse Fast FT / Fast FT (IFFT / FFT). This transform however has the drawback that it uses a rectangular window, which creates rather high side lobes. Moreover, the pulse shaping function used to modulate each subcarrier extends to infinity in the frequency domain this leads to high interference and lower performance levels. In this paper a new wavelet based OFDM system is proposed which uses dual tree complex wavelet transform (DTCWT) instead of FT. Simulation shows that the use of DTCWT outperforms the use of wavelet packet transform (WPT) and outperforms the use of FT in conventional OFDM system in term of reduction of PAPR. The complementary cumulative distribution function (CCDF) of the PAPR for the OFDM based on DTCWT show about 2 dB improvements over WPM and conventional OFDM.

KEY WORDS

OFDM, WPT, CWT, DTCWT, FFT, Multicarrier Modulation, PAPR.

I. INTRODUCTION

Since the early 1990s the WT and WPT have received more and more attention in modern communications and have been widely used in wireless communication [14]. A number of modulation schemes based on wavelets have been proposed [1 - 9]. The OFDM implemented by using IFFT and FFT have some problems. One of a major problem of this system is the high peak-to-average power ratio (PAPR). Due to this problem we look at other type of modulation to generate the carrier. Many authors are proposed WPT [15 - 20], WPT has a high degree of side lobe suppression and the loss of orthogonality leads to lesser inter symbol interference (ISI) and inter carrier interference (ICI) than in conventional OFDM system. [9]. In this paper we proposed a new wavelet based OFDM system which uses DTCWT instead of FFT. DTCWT has the same advantages as the WPT, but DTCWT produce better results of PAPR reduction than WPT. In the OFDM system based on DTCWT the FFT and IFFT are replaced by the DTCWT and the inverse DTCWT (IDTCWT) respectively.

This paper is organized as follows: In section II we discuss the complex wavelet modulation using DTCWT; in section III we discuss the OFDM system based on DTCWT; in section IV we discuss the PAPR in the OFDM system based on DTCWT; and we conclude this paper in section V by discussion the simulation results.

A. Discrete wavelet transform (DWT)

The DWT replaces the infinitely oscillating sinusoidal basis functions of the FT with a set of locally oscillating basis functions called wavelets. In the classical setting, the wavelets are stretched and shifted versions of a fundamental, real-valued band-pass wavelet $\psi(t)$. When carefully chosen and combined with shifts of a real-valued low-pass scaling function $\phi(t)$, they form an orthonormal basis expansion for one-dimensional (1-D) real-valued continuous-time signals. That is, any finite energy analog signal $x(t)$ can be decomposed in terms of wavelets and scaling functions via

$$x(t) = \sum_{n=-\infty}^{\infty} c(n)\phi(t-n) + \sum_{j=0}^{\infty}\sum_{n=-\infty}^{\infty} d(j,n)2^{j/2}\psi(2^j t - n) \qquad 1$$

The scaling coefficients $c(n)$ and the wavelet coefficients $d(j,n)$ are computed via the inner products

$$c(n) = \int_{-\infty}^{\infty} x(t)\phi(t-n)dt \qquad 2$$

$$d(j,n) = \int_{-\infty}^{\infty} x(t)\psi(2^j t - n)dt \qquad 3$$

They provide a time-frequency analysis of the signal by measuring its frequency content (controlled by the scale factor j) at different times (controlled by the time shift n). There exists a very efficient, linear time complexity algorithm to compute the coefficients $c(n)$ and $d(j,n)$ from a fine-scale representation of the signal (often simply N samples) and vice versa based on two octave-band, discrete-time filter banks (FBs) that recursively apply a discrete-time low-pass filter $h_0(n)$, a high-pass filter $h_1(n)$, and upsampling and downsampling operations. Fig. 1 show the FB trees implementation, the analysis (forward) and synthesis (inverse) DWT.

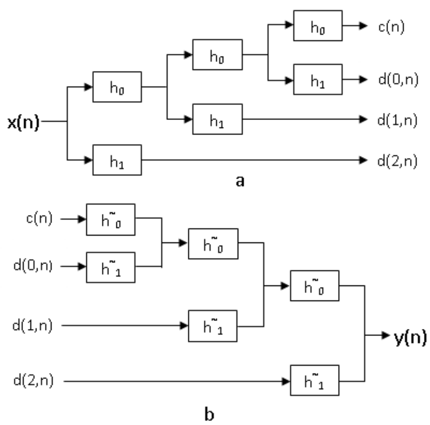

Fig. 1. Filter bank trees implementing the (a) analysis (forward) and (b) synthesis (inverse) DWT.

B. Dual tree complex wavelet transform DTCWT

Complex wavelet transform (CWT) is applied perfectly to digital image processing. Kingsbury [21], [24 - 27] introduced and made a concrete description of DTCWT. The DTCWT inspires a new filter pairs ($h_0(n)$, $h_1(n)$ the low-pass/high-pass filter pair for the upper FB respectively) and ($g_0(n)$, $g_1(n)$g$_1$ the low-pass/high-pass filter pair for the lower FB respectively) are used to define the sequence of $\psi(t)$ and $\phi(t)$ as follows

$$\psi_h(t) = \sqrt{2}\sum_n h_1(n)\phi_h(t) \qquad 4$$

$$\phi_h(t) = \sqrt{2}\sum_n h_0(n)\phi_h(t) \qquad 5$$

Where $h_1(n) = (-1)^n h_0(d-n)$, $\psi_g(t)$, $\phi_g(t)$ and $g_1(n)$ are defined similarly. The two real wavelets associated with each of the two real transform are $\psi_h(t)$ and $\psi_g(t)$. The filters are designed so that $\psi_g(t)$ is approximately the Hilbert transform of $\psi_h(t)$. [$\psi_g(t) = H\{\psi_h(t)\}$].

The DTCWT employs two real DWTs; the upper one gives the real part of the transform while the lower one gives the imaginary part. The analysis and the synthesis FBs used to implement the DTCWT and its inverse are illustrated in fig. 2 and fig. 3.

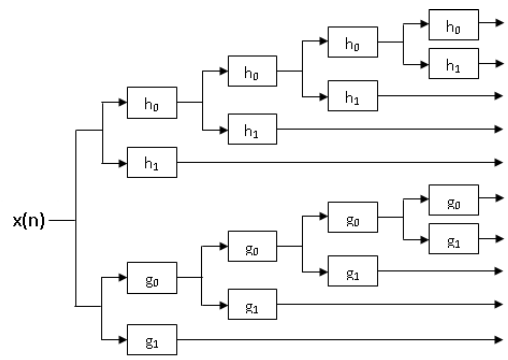

Fig. 2. Analysis FB for the dual tree discrete CWT (DTDCWT).

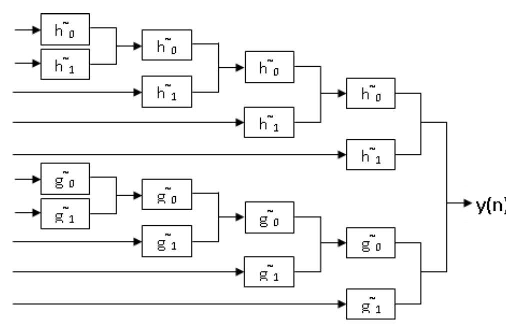

Fig. 3. Synthesis FB for the dual tree discrete CWT (DTDCWT).

If the two real DWTs are represented by the square matrices F_h for the upper part and F_g for the lower part, then the DTCWT can be represented by the following form.

$$F_c = \frac{1}{\sqrt{2}}\begin{bmatrix} I & jI \\ I & -jI \end{bmatrix} \cdot \begin{bmatrix} F_h \\ F_g \end{bmatrix} \qquad 6$$

And the IDTCWT is given by

$$F_c^{-1} = \frac{1}{\sqrt{2}}\begin{bmatrix} F_h^{-1} & F_g^{-1} \end{bmatrix}^{-1} \cdot \begin{bmatrix} I & I \\ -jI & jI \end{bmatrix} \qquad 7$$

Note that the complex sum/difference matrix in (6) is unitary (its conjugate transpose is its inverse).

$$\frac{1}{\sqrt{2}}\begin{bmatrix} I & jI \\ I & -jI \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ -jI & jI \end{bmatrix} = I \qquad 8$$

Note that the identity matrix on the right-hand side of (8) is twice the size of those on the left-hand side. Therefore if the two real DWTs are orthonormal transforms, then the DTCWT satisfies $F_c^*.F_c = I$, where * denotes the conjugate transpose.

C. peak to average power ratio (PAPR)

The PAPR of the baseband transmitted signal $x(t)$ is defined as the ratio of the peak power (P_{peak} = max $\{|x(t)|^2\}$); i.e., the maximum power of the transmitted signal over the average power (P_{ave} = E $\{|x(t)|^2\}$). In digital implementations of communications transceivers, rather than using the continuous time signal $x(t)$ in PAPR computation, we instead work with $x[n]$, the discrete time samples of x(t), provided that an oversampling factor of at least 4 is used. PAPR is then expressed as [22]:

$$PAPR = \frac{\max\{|x[n]^2|\}}{\mathrm{E}\{|x[n]^2|\}} \quad 9$$

where $E\{.\}$ denotes ensemble average calculated over the duration of the OFDM, WPM, or complex wavelet modulation symbol (CWM) (using DTCWT).

In both OFDM and WPM systems, the signal going into the channel $x(t)$ is a sum of random symbols modulating orthogonal basis functions. Based on the central limit theorem (CLT), it is claimed that $x(t)$ is complex Gaussian and its envelope follows a Rayleigh distribution. This implies a large PAPR. A high PAPR of the transmitted signals demands a very linear transmission path and limits the practical deployment of low-cost non-linear power amplifiers forcing them to operate with a reduced power efficiency. Driving amplifiers operating close to saturation with signals of high PAPR results in the generation of unwanted spectral energy both in-band and out-of-band, which in turn reduces the systems bit error rate (BER) performance and gives rise to adjacent channel interference (ACI), respectively.

In this work, the performance of the OFDM based on DTCWT in PAPR reduction is demonstrated through the CCDF of PAPR, which is a performance metric independent of the transmitter amplifier.

Given the reference level $PAPR_0 > 0$, the probability of a PAPR being higher than the reference value is the CCDF and is expressed as follows [23]:

$$CCDF(PAPR_0) = P_r\{PAPR > PAPR_0\} \quad 10$$

II. COMPLEX WAVELET MODULATION (CWM)

In the CWM using DTCWT, an IDTCWT or synthesis FB, which is based on DTCWT basis function, is used for multiplexing. Wavelet modulation has the following advantage: high spectral efficiency because of low out-of-band energy (low side lobes), and providing robustness with regard to ICI. Moreover wavelet modulation is able to decompose time-frequency plane flexibility by arranging FB construction. In addition, using FFT in the OFDM system has only resolution in the frequency domain while using WT in the OFDM system gives a resolution in both time and frequency domains. Wavelet modulation achieves a good performance in a tone/impulse interference environment, because of the inherent flexibility. A functional block diagram of the CWM using DTCWT is shown in fig. 4.

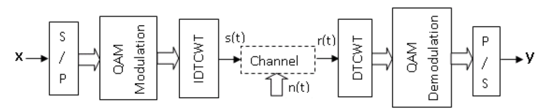

Fig. 4. Complex wavelet modulation functional block diagram using DTCWT

Similar to OFDM system, at the transmitter an IDTCWT block is used in place of the IFFT block, while at the receiver a DTCWT block is used in the place of the FFT block. IDTCWT and DTCWT are works in a similar fashion to an IFFT and FFT blocks respectively.

III. DTCWT BASED OFDM SYSTEM

The IDTCWT takes as the input QAM or PSK symbols and outputs them in parallel time-frequencies "subcarriers." On the CWM visualized in the left side of fig. 4 as the synthesis process, it can be shown that the transmitted signal, $s[n]$ is synthesized as

$$s(t) = \sum_{m=1}^{M} \sum_{n=0}^{+\infty} a_m(n)\, \psi_{k,m}(t - nT) \quad 11$$

Where, $a_m(n)$ is the transmitted symbol, $\psi_{k,m}(t - nT)$ is complex wavelet function for the m^{th} subcarrier, T is the time period, and M is the number of active subcarriers.

$$\psi(t) = \psi_h(t) + j\, \psi_g(t) \quad 12$$

The DTCWT at the receiver recovers the transmitted symbols (a_m) through the analysis formula exploiting orthogonality properties of DTCWT and is schematically represented in the right side of fig. 4.

IV. PAPR IN OFDM BASED ON DTCWT

The DTCWM synthesis (11) is very similar to that of conventional OFDM where the discrete function $\psi(n)$ are replaced by the N finite duration complex exponential $\exp(j2\pi\frac{n}{N}k)$. Specifically, the OFDM transmitter is given by

$$x^m[n] = \sum_{k=0}^{N-1} a_k^m e^{2\pi nk/N} \qquad 13$$

Where the input is the m^{th} frame of N QAM symbols $(a_0^m, a_1^m, \ldots, a_{N-1}^m)$. In other hand (11) is also very similar to that of WPM where the DTCW function is replaced by the wavelet packet function. From CLT, $x[n]$ is complex Gaussian distributed, and the sequence $x[n]$ has high PAPR. The envelope of the conventional OFDM, WPM, and OFDM based on DTCWT are illustrated in fig. 5 we see that a similar behavior of the transmitted envelope in conventional OFDM and WPM systems is illustrated in the simulations results presented in fig. 5.

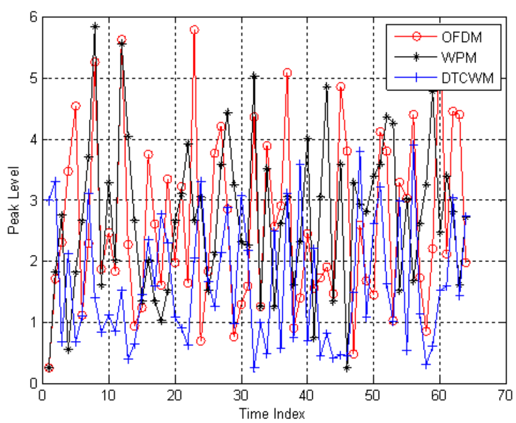

Fig. 5. The envelope of the conventional OFDM, WPM, and OFDM based on DTCWT outputs

V. SIMULATION RESULTS

The results given in this section compare the PAPR in OFDM based on DTCWT, with that for conventional OFDM, and WPM. To be able to make a fair comparison, the same simulation parameters are used. The simulation parameters are documented as follows:

Modulation type is 16-QAM; the number of subcarriers is 64 subcarriers; a wavelet packet base is Daubechies-1 (DAUB-1); PAPR threshold is 2dB; shaping filter is Raised Cosine ($\propto = 0.01$, upsamler = 5.)

The results are quantified using CCDF for PAPR and they are shown in fig. 6. The CCDF plots shows that the OFDM based on DTCWT offers the best PAPR performance without using any reduction techniques. Fig. 6 shows about 2 dB improvement in OFDM based on DTCWT over the conventional OFDM and WPM systems.

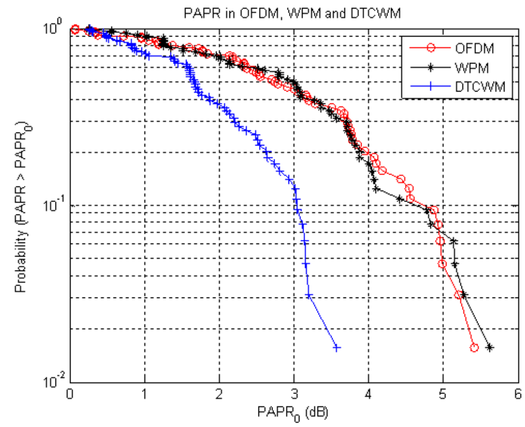

Fig. 6. CCDF results for OFDM based on DTCWT

CONCLUSION

In this paper a new OFDM scheme is proposed. This new scheme uses DTCWT instead of FT, where, FFT and IFFT are replaced by DTCWT and IDTCWT respectively. Simulation results show the new scheme gives about 2dB improvement in PAPR reduction over the conventional OFDM and WPM systems.

From the results we found the conventional OFDM and WPM systems are produce same results in PAPR reduction. Also the transmitted envelope for the conventional OFDM and WPM systems are illustrated similar behavior.

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